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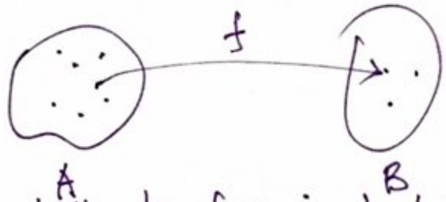
- Funzioni Booleane

<u>Def</u> 1. Una variabile booleana è una "quantità" che può assumere ~~[illegible]~~ solamente 2 valori

$$\{0, 1\}$$

<u>Def</u> 2. Una funzione Booleana di ~~[illegible]~~ una variabile booleana è

$$f : \{0,1\} \rightarrow \{0,1\}$$



Quali sono tutte le funzioni booleane di 1 variabile booleana

$0 \rightarrow 0$

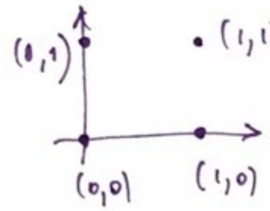
1

0 0
0 1
1 0
1 1

$f(0)$	$f(1)$	
0	0	
1	1	
0	1	identità
1	0	negazione

Funzioni Booleane di 2 variabili booleane

$$f: \{0,1\} \times \{0,1\} \longrightarrow \{0,1\}$$



Quante sono le funzioni booleane di 2 variabili (booleane)

$$f(0,0),\ f(0,1),\ f(1,0),\ f(1,1)$$
$$2 \cdot 2 \cdot 2 \cdot 2 = 2^4 = 16$$

Esercizio Quante sono le funzioni booleane di n variabili booleane

R1. 2^n /No

R2. 2^{2^n} se $n=2$ $2^{2^2} = 2^4 = 16$

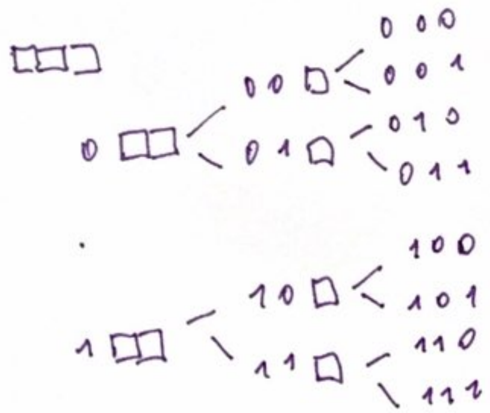
la funzione è specificata dal valore delle n-uple

$$\underbrace{\overset{x_1\,x_2\;\;\;x_n}{00\cdots0}}_{n} \quad 00\cdots1 \quad 00\cdots01 \quad \cdots\cdots \quad 11\cdots1$$
$$\underbrace{2 \qquad 2 \qquad 2 \qquad \cdots \qquad 2}_{2^{(2^n)}}$$

Contiamo tutte le possibili n-uple fatte di 0 e 1

$$\overset{2}{\underset{1}{\square}}\overset{2}{\underset{2}{\square}} \cdots \overset{2}{\underset{n}{\square}} \qquad \underbrace{2 \cdot 2 \cdot 2 \cdots}_{n \text{ volte}} 2 = 2^n$$

$n=3$



$8 = 2^3$

	$f(0,0)$	$f(0,1)$	$f(1,0)$	$f(1,1)$
1	0	0	0	0
2	0	0	0	1
3	0	0	1	0
4	0	0	1	1
5	0	1	0	0
6	0	1	0	1
7	0	1	1	0
8	0	1	1	1
9	1	0	0	0
10	1	0	0	1
11	1	0	1	0
12	1	0	1	1
13	1	1	0	0
14	1	1	0	1
15	1	1	1	0

16 1 1 1 1

$f(0,0) = 0$

$f(0,1) = 1$

$f(1,0) = 1$ Xor

$f(1,1) = 0$

$f(x,y) = x \{\} y$

$f_{Xor}(x,y) = x \oplus y$

<u>Esercizio</u> Dimostrare che

- $(x \vee y) \wedge z = (x \wedge z) \vee (y \wedge z)$
- $(x \wedge y) \vee z = (x \vee z) \wedge (y \vee z)$

x	y	z	$(x \vee y) \wedge z$	$(x \wedge z) \vee (y \wedge z)$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	1	1
1	0	0	0	0
1	0	1	1	1
1	1	0	0	0
1	1	1	1	1

Table 1

THE SIXTEEN LOGICAL OPERATIONS ON TWO VARIABLES

Truth table	New and old notation(s)	Operator symbol $\circ$	Name(s)
0000	0	$\perp$	Contradiction; falsehood; antilogy; constant 0
0001	$xy,\ x\wedge y,\ x\mathbin{\&} y$	$\wedge$	Conjunction; and
0010	$x\wedge\bar y,\ x\not\supset y,\ [x>y],\ x\mathbin{\dot-}y$	$\bar\supset$	Nonimplication; difference; but not
0011	x	$\llcorner$	Left projection; first dictator
0100	$\bar x\wedge y,\ x\not\subset y,\ [x<y],\ y\mathbin{\dot-}x$	$\bar\subset$	Converse nonimplication; not ... but
0101	y	R	Right projection; second dictator
0110	$x\oplus y,\ x\not\equiv y,\ x\hat{\ }y$	$\oplus$	Exclusive disjunction; nonequivalence; “xor”
0111	$x\vee y,\ x\mid y$	$\vee$	(Inclusive) disjunction; or; and/or
1000	$\bar x\wedge\bar y,\ \overline{x\vee y},\ x\mathbin{\bar\vee}y,\ x\downarrow y$	$\bar\vee$	Nondisjunction; joint denial; neither ... nor
1001	$x\equiv y,\ x\leftrightarrow y,\ x\Leftrightarrow y$	$\equiv$	Equivalence; if and only if; “iff”
1010	$\bar y,\ \neg y,\ !y,\ {\sim}y$	$\bar{\mathsf{R}}$	Right complementation
1011	$x\vee\bar y,\ x\subset y,\ x\Leftarrow y,\ [x\geq y],\ x^y$	$\subset$	Converse implication; if
1100	$\bar x,\ \neg x,\ !x,\ {\sim}x$	$\bar\llcorner$	Left complementation
1101	$\bar x\vee y,\ x\supset y,\ x\Rightarrow y,\ [x\leq y],\ y^x$	$\supset$	Implication; only if; if ... then
1110	$\bar x\vee\bar y,\ \overline{x\wedge y},\ x\mathbin{\bar\wedge}y,\ x\mid y$	$\bar\wedge$	Nonconjunction; not both ... and; “nand”
1111	1	$\top$	Affirmation; validity; tautology; constant 1

it by the truth table shown in Table 1, which states in particular that the implication is true when both x and y are false. Much of this early work has been lost, but there are passages in the works of Galen (2nd century A.D.) that refer to both inclusive and exclusive disjunction of propositions. [See I. M. Bocheński, *Formale Logik* (1956), English translation by Ivo Thomas (1961), for an excellent survey of the development of logic from ancient times up to the 20th century.]

A function of two variables is often written $x\circ y$ instead of $f(x,y)$, using some appropriate operator symbol $\circ$. Table 1 shows the sixteen operator symbols that we shall adopt for Boolean functions of two variables; for example, $\perp$ symbolizes the function whose truth table is 0000, $\wedge$ is the symbol for 0001, $\bar\supset$ is the symbol for 0010, and so on. We have $x\perp y=0$, $x\wedge y=xy$, $x\mathbin{\bar\supset}y=x\mathbin{\dot-}y$, $x\mathbin{\llcorner}y=x$, $\ldots$, $x\mathbin{\bar\wedge}y=\bar x\vee\bar y$, $x\top y=1$.

Of course the operations in Table 1 aren't all of equal importance. For example, the first and last cases are trivial, since they have a constant value independent of x and y. Four of them are functions of x alone or y alone. We write $\bar x$ for $1-x$, the *complement* of x.

The four operations whose truth table contains just a single 1 are easily expressed in terms of the AND operator $\wedge$, namely $x\wedge y$, $x\wedge\bar y$, $\bar x\wedge y$, $\bar x\wedge\bar y$. Those with three 1s are easily written in terms of the OR operator $\vee$, namely $x\vee y$, $x\vee\bar y$, $\bar x\vee y$, $\bar x\vee\bar y$. The basic functions $x\wedge y$ and $x\vee y$ have proved to be more useful in practice than their complemented or half-complemented cousins, although the NOR and NAND operations $x\mathbin{\bar\vee}y=\bar x\wedge\bar y$ and $x\mathbin{\bar\wedge}y=\bar x\vee\bar y$ are also of interest because they are easily implemented in transistor circuits.

Esercizio Vero o Falso?

(a) $(x \oplus y) \vee z = (x \vee z) \oplus (y \vee z)$

(b) $(w \oplus x \oplus y) \vee z = (w \vee z) \oplus (x \vee z) \oplus (y \vee z)$

(c) $(x \oplus y) \vee (y \oplus z) = (x \oplus z) \vee (y \oplus z)$

~~Proof~~

Soluzione (b) Posso scegliere in (b) $z = 0$

$$w \oplus x \oplus y = w \oplus x \oplus y$$

Pongo $z = 1$ in (b)

$$1 = 1$$

$$\underbrace{\underbrace{1 \oplus 1}_{0} \oplus 1}_{1}$$

$x \vee 0 = x$

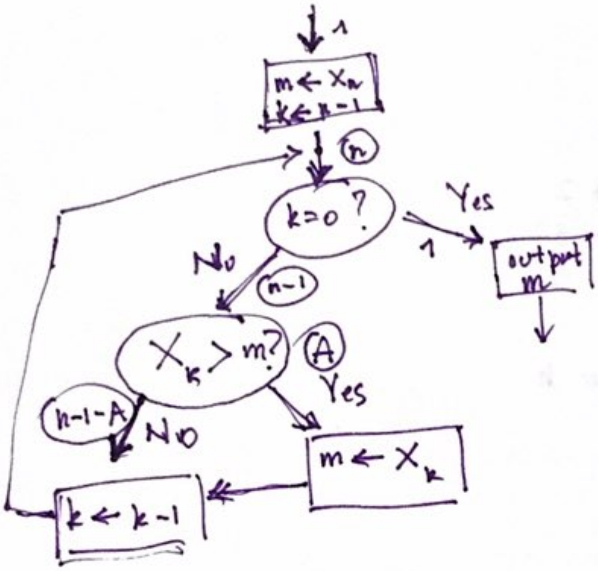
$x \vee 1 = 1$

Analisi degli Algoritmi

"Find the maximum" (Trovare il massimo)

$X_1, X_2, \dots, X_n$

$a \leftarrow b$



$m \quad \cancel{0.5} \rightarrow \boxed{4.1}$

$k \quad \cancel{3} \rightarrow \cancel{2} \rightarrow \cancel{1} \rightarrow 0$

$n = 4 \qquad X_1, \; X_2, \; X_3, \; X_4$

$\qquad 2.3 \quad 3.2 \quad \underline{4.1} \quad 0.5$

m 0.5

k 3

A = # volte che "scambio il massimo"

$\min A = 0 \qquad (\text{se } X_n = \max_{i \in \{1,\dots,n\}} X_i)$

$\max A = n-1 \qquad (\text{se } X_1 > X_2 > X_3 > \dots > X_n)$

$\text{ave } A = ?$

$n=3$ #/

			A
1	2	3	0
1	3	2	1
2	1	3	0 ←
2	3	1	1
3	1	2	1
3	2	1	2

8	4	7
3	1	2

\# sequenze dove $A=0 \to 2$

\# sequenze dove $A=1 \to 3$

\# sequenze dove $A=2 \to 1$

P_{nk} = # sequenze lunghe n dove $A=k$

$P_{30}=2$ $\quad \frac{2}{6}$

$P_{31}=3$

$\vdots$

(probabilità che $A=k$) = $\frac{\text{\# numero di sequenze con } A=k}{\text{\# sequenze}} = p_{nk}$

$$\boxed{p_{nk}=\frac{P_{nk}}{n!}}$$

$$\boxed{EA=\sum_k \underbrace{p_{nk}}_{\Pr(A=k)} \cdot k}$$

$n! = n(n-1)\cdots 3\cdot2\cdot1$

$n(n-1)(n-2)\cdots 1 = n!$

$$EA = p_{30}\cdot0 + p_{31}\cdot1 + p_{32}\cdot2 + 0$$

$$= \frac{2}{6}\cdot0 + \frac{3}{6}\cdot1 + \frac{1}{6}\cdot2 = \frac{1}{2}+\frac{1}{3} = \frac{3+2}{6} = \frac{5}{6}$$

		A
1	2 3 4	0
	2 4 3	1
	3 2 4	0
	3 4 2	1
	4 2 3	1
	4 3 2	2
2	1 3 4	0
	1 4 3	1
	3 1 4	0
	3 4 1	1
	4 1 3	1
	4 3 1	2
3	1 2 4	0
	1 4 2	1
	2 1 4	0
	2 4 1	1
	4 1 2	1
	4 2 1	2
4	1 2 3	1
	1 3 2	2
	2 1 3	1
	2 3 1	2
	3 1 2	2
	3 2 1	3