COVID-19 Italian and European pandemic: a SEIR model with undetected fraction and mobility-dependent transmission rate

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The SIR compartmental model

Original version: Kermack and McKendrick 1927

S = number of susceptibles in a population

I = number of infected in a population

R = number of recovered in a population

Average contacts per day per person

Probability of infection due to S-I contact

$$\frac{dS}{dt} = -\lambda \frac{I}{N} S \rho$$

$$\frac{dI}{dt} = \lambda \frac{I}{N} S \rho - \gamma I$$

$$\frac{dR}{dt} = \gamma I$$

Inverse of recovery time

The SIR compartmental model

Transmission rate and basic reproduction number

$$\beta = \lambda \ \rho$$ Transmission rate

$$R_0 = \frac{\beta}{\gamma}$$ Basic reproduction number

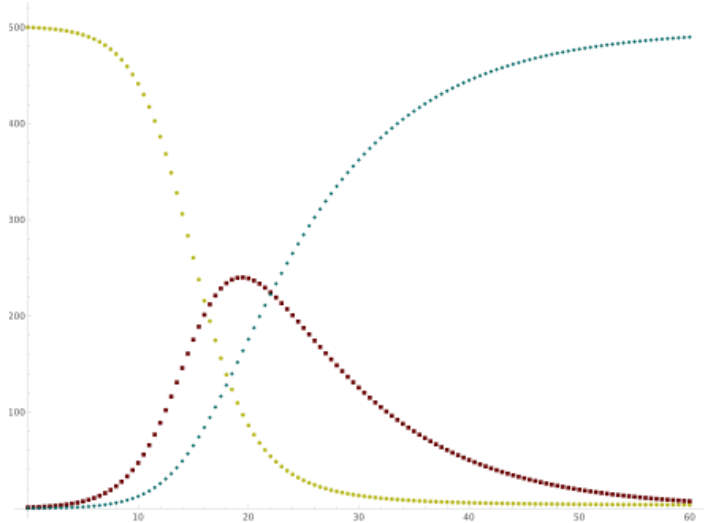
$$S \approx N$$

$$\begin{aligned} \frac{dS(t)}{dt} &= -\beta\, I(t) \\ \frac{dI(t)}{dt} &= (\beta - \gamma)\, I(t) \\ \frac{dR(t)}{dt} &= \gamma\, I(t) \end{aligned}$$

$$I(t) = I_0\, e^{(\beta-\gamma)t}$$

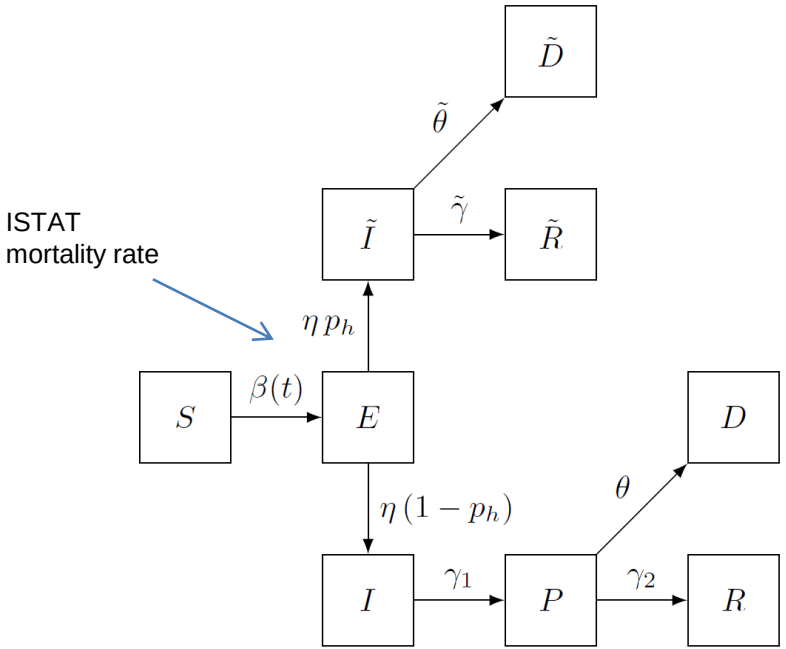
$$R_0 > 1$$

$$R_0 < 1$$



The SEIR modified model

1) Undetected fraction of infected



$$\frac{dS}{dt} = -\beta(t)\, S(t)\, \frac{I(t)+\tilde{I}(t)}{N} \quad (1)$$

$$\frac{dE}{dt} = \beta(t)\, S(t)\, \frac{I(t)+\tilde{I}(t)}{N} - \eta E \quad (2)$$

$$\frac{dI}{dt} = (1-p_h)\,\eta\, E - \gamma_1\, I(t) \quad (3)$$

$$\frac{dP}{dt} = \gamma_1\, I(t) - (1-p_d)\,\gamma_2\, P(t) - p_d\,\theta P(t) \quad (4)$$

$$\frac{dR}{dt} = (1-p_d)\,\gamma_2\, P(t) \quad (5)$$

$$\frac{dD}{dt} = p_d\,\theta\, P(t) \quad (6)$$

$$\frac{d\tilde{I}}{dt} = p_h\,\eta\, E(t) - (1-\tilde{p}_d)\,\tilde{\gamma}\,\tilde{I}(t) - \tilde{p}_d\,\tilde{\theta}\,\tilde{I}(t) \quad (7)$$

$$\frac{d\tilde{R}}{dt} = (1-\tilde{p}_d)\,\tilde{\gamma}\,\tilde{I}(t) \quad (8)$$

$$\frac{d\tilde{D}}{dt} = \tilde{p}_d\,\tilde{\theta}\,\tilde{I}(t) \quad (9)$$

```
seir = partial(SEIR, beta=beta_
sol_ = solve_ivp(seir, [0, t_fi
```

The SEIR modified model

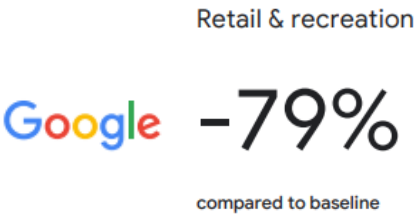
2) Time dependent transmission rate

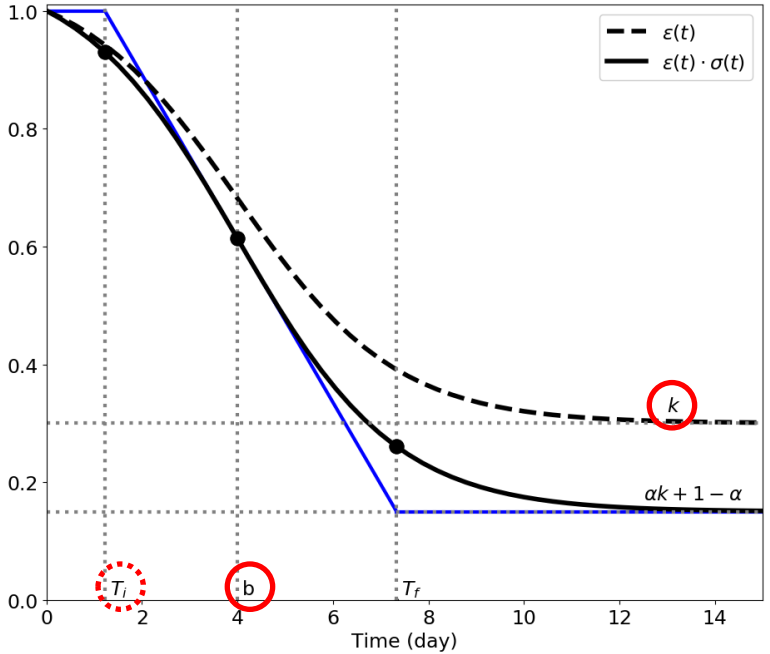
$$\lambda(t) = \lambda_0\,\epsilon(t) \qquad \rho(t) = \rho_0\,\sigma(t)$$

$$\epsilon(t) = \frac{C(a,b)}{1 + e^{a\,(t-b)}} + k$$

$$\epsilon(t)\cdot\sigma(t) := \alpha\left(\frac{C}{1 + e^{a\,(t-b)}} + k\right) + (1-\alpha)$$

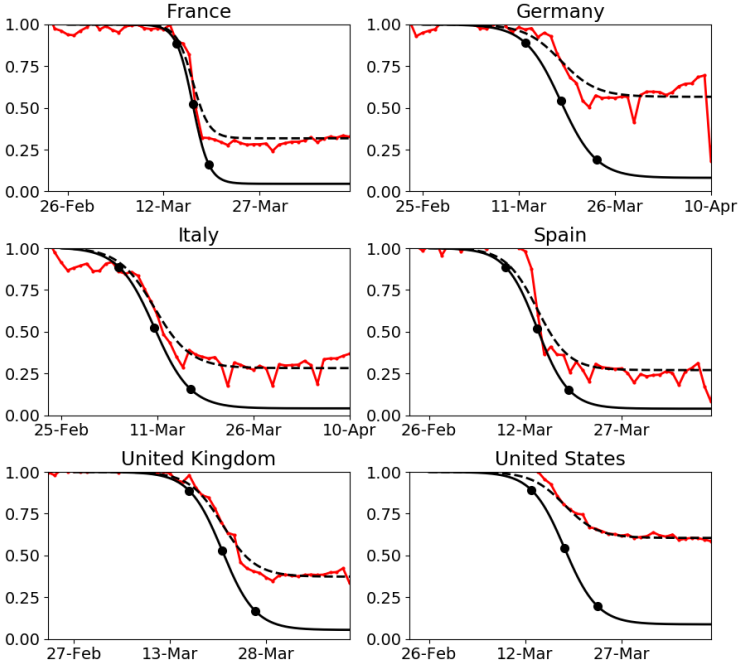
$$\beta(t) = \lambda_0\,\rho_0\,\epsilon(t)\,\sigma(t)$$

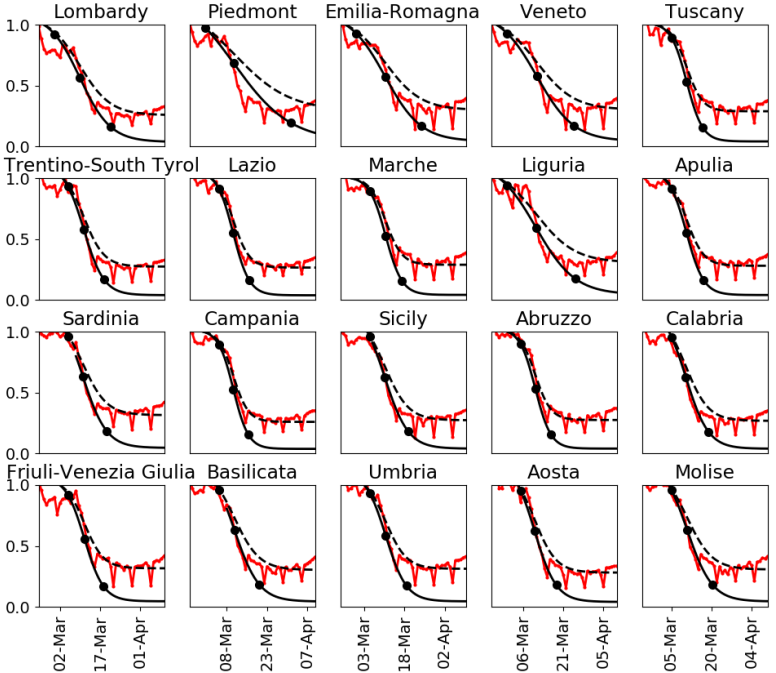




The SEIR modified model

Mobility fitting





Experimetal results
Remarks

1) Fitting parameters need to have an epidemical meaning

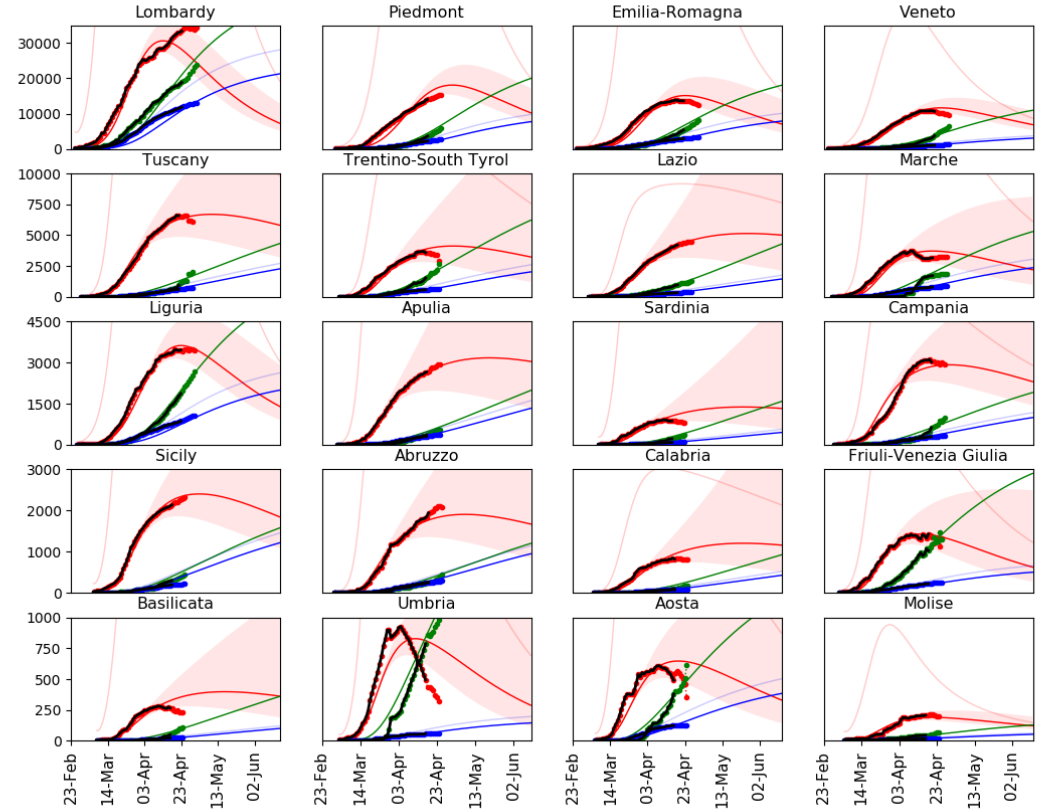
- Initial beta depending on the local mobilility

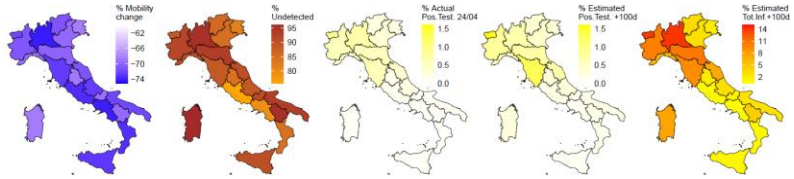
- The reporting time: from recovery to second negative test

2) Overfitting

Experimental results

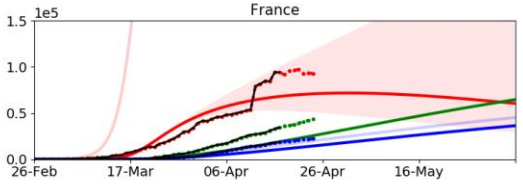
1) Italian regions

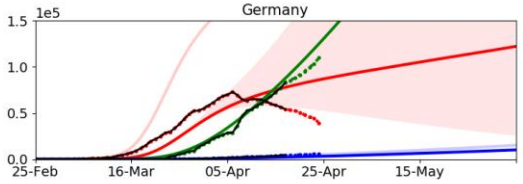


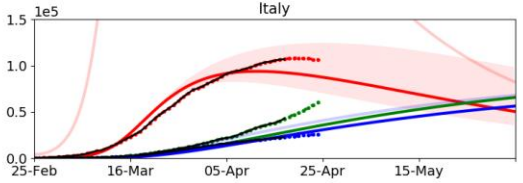


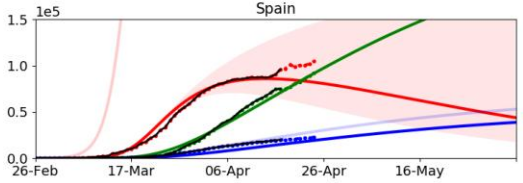
Regions	N (M)	D_w/D_b	%U	β(0)	$R(0)_b$	$R(0)_w$	R(0)	$1/\gamma_2$	MAPE (%)	Actual Pos.Test. 24/4/20 *1000	Est. Pos. Test. 24/4/20 *1000 (CI)	Est. Tot. Inf. 24/4/20 *1000 (CI)	Popol. Tot. Inf. 24/4/20 % (CI)
Abruzzo	1.31	0.30	89	1.96	9.4	19.7	18.5	100	3.8	2.8	2.7 (2.0 - 4.4)	26 (22 - 38)	1.6 (1.3 - 2.4)
Aosta	0.13	0.00	89	1.77	8.5	17.8	18.5	33	12.3	1.1	1.3 (1.0 - 1.9)	12 (11 - 15)	2.0 (1.8 - 2.5)
Apulia	4.03	1.40	93	2.07	9.9	20.8	20	100	2.8	3.9	4.0 (2.9 - 6.9)	63 (51 - 95)	0.5 (0.4 - 0.7)
Basilicata	0.56	5.00	95	1.47	7	14.7	17.8	75	31.4	0.4	0.5 (0.4 - 0.7)	11 (10 - 14)	0.4 (0.3 - 0.5)
Calabria	1.95	0.26	85	2.20	10.5	22	21.2	82	28.0	1.1	1.4 (1.1 - 2.3)	10 (8 - 15)	9.6 (9.3 - 10.5)
Campania	5.80	0.00	79	1.79	8.6	17.9	16	100	3.2	4.3	4.1 (3.3 – 6.0)	21 (19 - 26)	1.4 (1.3 - 1.8)
Emilia-Romagna	4.46	0.65	93	1.10	5.3	11	10.6	42	13.2	24.0	27.3 (23.1 - 35.5)	426 (414 - 470)	0.5 (0.4 - 0.8)
Friuli-Venezia Giulia	1.22	0.00	82	1.53	7.3	15.4	13.9	29	3.7	2.9	3.0 (2.4 - 4.5)	17 (16 - 22)	5.0 (4.9 - 5.4)
Lazio	5.88	0.00	76	2.50	12	25.1	21.9	77	2.7	6.1	6.4 (4.7 - 11.4)	29 (24 - 45)	13.4 (13.6 - 13.9)
Liguria	1.55	0.05	90	1.01	4.9	10.2	9.6	33	0.9	7.2	7.1 (6.1 - 9.1)	77 (76 - 83)	5.4 (4.6 - 7.2)
Lombardy	10.06	0.58	95	0.97	4.6	9.7	9.5	32	8.6	71.3	63.6 (55.8 - 78.2)	1353 (1370 - 1395)	1.0 (1.0 – 1.0)
Marche	1.53	0.10	91	1.77	8.5	17.8	16.9	40	18.6	6.0	7.3 (5.5 - 11.3)	82 (70 - 110)	8.5 (8.7 - 8.6)
Molise	0.31	1.84	91	0.90	4.3	9	8.9	100	7.9	0.3	0.3 (0.2 - 0.3)	3 (3 - 3)	3.1 (2.6 - 4.1)
Piedmont	4.36	0.69	92	0.93	4.4	9.3	9.3	47	12.4	23.8	26.9 (23.9 - 32.2)	369 (377 - 373)	0.7 (0.7 - 0.9)
Sardinia	1.64	4.65	96	1.62	7.8	16.2	19.9	57	29.7	1.3	1.7 (1.3 - 2.7)	50 (43 - 68)	5.0 (4.1 - 7.3)
Sicily	5.00	0.49	90	1.53	7.3	15.3	16	98	12.4	3.0	3.3 (2.7 - 4.7)	37 (34 - 45)	2.3 (2 - 3.1)
Trentino-South. Tyrol	1.07	0.83	86	2.26	10.8	22.7	21	42	9.7	6.2	7.0 (5.1 - 11.7)	53 (44 - 78)	1.5 (1.3 - 1.8)
Tuscany	3.73	0.00	89	1.74	8.4	17.5	16.5	100	2.2	8.9	9.2 (7 - 14.3)	86 (74 - 116)	2.5 (2.6 - 2.6)
Umbria	0.88	1.23	84	1.48	7.1	14.9	13.6	23	34.8	1.4	1.9 (1.5 - 2.8)	13 (12 - 16)	2.1 (1.8 - 2.8)
Veneto	4.91	0.57	85	0.91	4.4	9.1	8.4	61	5.3	17.2	17.9 (16 - 21.6)	125 (128 - 127)	0.7 (0.6 - 1.2)

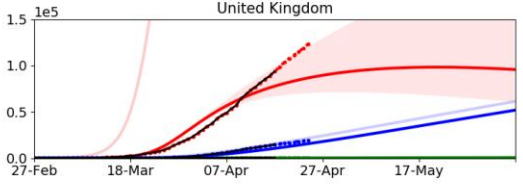
Experimetal results
2) Countries

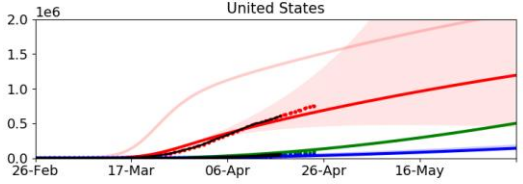








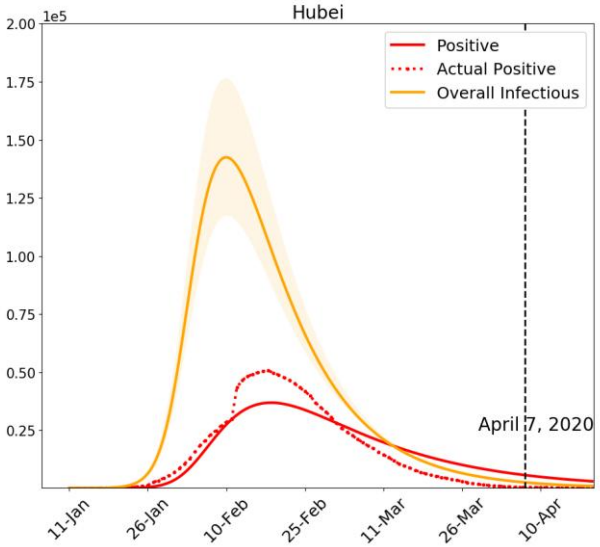


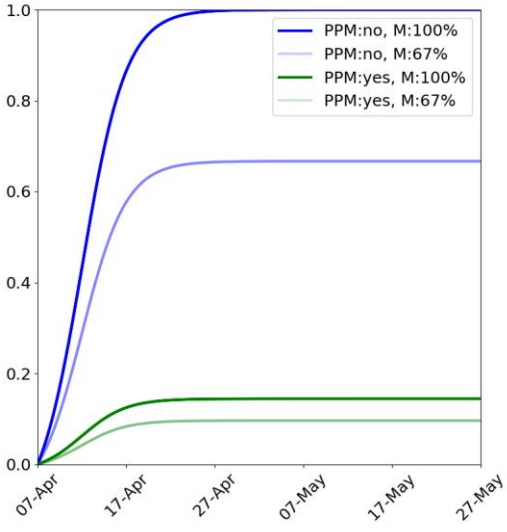


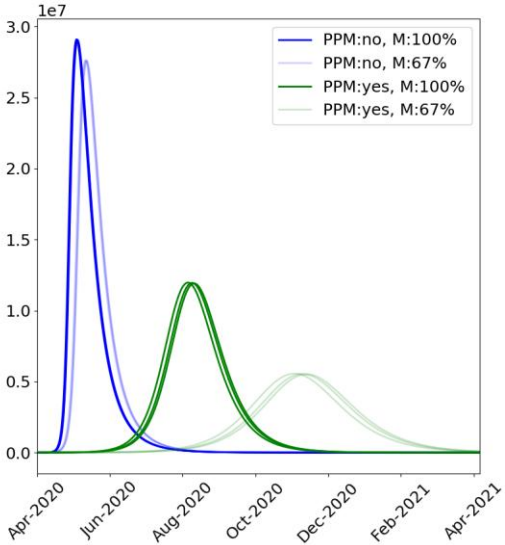
Regions	N (M)	D_H/D_0	%U	β(0)	$R(0)_0$	$R(0)_H$	R(0)	$1/\gamma_2$	MAPE (%)	Actual Pos.Test. 24/4/20 *1000	Est. Pos. Test. 24/4/20 *1000 (CI)	Est. Tot. Inf. 24/4/20 *1000 (CI)	Popol. Tot. Inf. 24/4/20 % (CI)
France	66.99	0.53	91	1.71	8.2	17.1	16.3	66	31.8	158.6	110.7 (84.5 - 170.6)	1376 (1186 - 1852)	3.7 (3.6 - 4.1)
Germany	83.02	0.53	60	1.72	8.3	17.3	13.6	15	30.0	155.0	214.0 (159.8 - 396.4)	610 (501 - 1020)	4.9 (4.3 - 6.5)
Italy	60.32	0.53	93	1.17	5.6	11.7	11.2	81	18.8	193.0	153.0 (129.2 - 202.4)	2212 (2149 - 2476)	2.2 (2.1 - 2.6)
Spain	46.94	0.53	91	1.92	9.2	19.3	18.4	26	10.2	219.8	193.1 (148.8 - 295.6)	2294 (2003 - 3048)	1.2 (0.9 - 1.8)
United Kingdom	66.65	0.53	92	1.27	6.1	12.7	12.2	10000†	24.9	143.5	102.4 (84.3 - 139.5)	1474 (1374 - 1739)	3.0 (2.35 - 5.41)
United States	328.20	0.53	76	1.38	6.6	13.9	12.1	100	8.5	905.4	814.1 (573.6 – 1415.0)	3785 (2949 - 5911)	2.44 (1.2 - 9.67)

Experimental results

3) Hubei



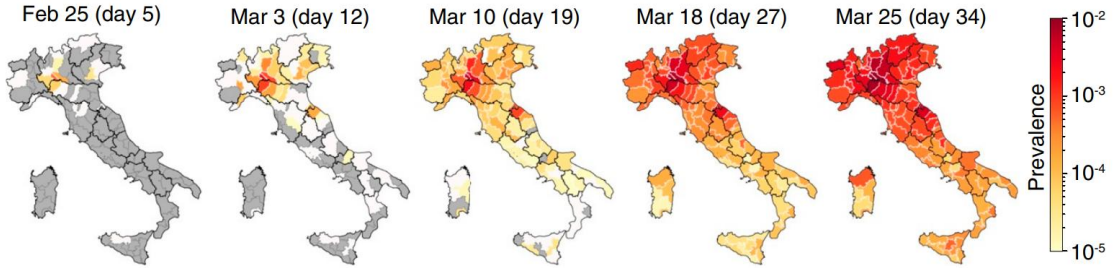




Limitations & Perspectives

Limitations

1. Actual Dataset of I, R and D
2. Deterministic model
3. Demographic changes of the population, stratification for age etc…
4. Spatial homogeneity hypothesis



Limitations & Perspectives

Perspectives

Lack of historical data due to their outlier nature

1) NLP to analyze web traces (BlueDot) for Social control / Warning

2) AI techniques for Diagnosis and Treatment

3) GNN for spatial distribution

Thanks for your attention